

## Chapter 0

### Extended anti-de Sitter Hypergravity in $2 + 1$ Dimensions and Hypersymmetry Bounds

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In a recent paper (JHEP **1508** (2015) 021), we have investigated hypersymmetry bounds in the context of simple anti-de Sitter hypergravity in  $2+1$  dimensions. We showed that these bounds involved non linearly the spin-2 and spin-4 charges, and were saturated by a class of extremal black holes, which are  $\frac{1}{4}$ -hypersymmetric. We continue the analysis here by considering  $(M, N)$ -extended anti-de Sitter hypergravity models, based on the superalgebra  $osp(M|4) \oplus osp(N|4)$ . The asymptotic symmetry superalgebra is then the direct sum of two-copies of a  $W$ -superalgebra that contains  $so(M)$  (or  $so(N)$ ) Kac-Moody currents of conformal weight 1, fermionic generators of conformal weight  $5/2$  and bosonic generators of conformal weight 4 in addition to the Virasoro generators. The nonlinear hypersymmetry bounds on the conserved charges are derived and shown to be saturated by a class of extreme hypersymmetric black holes which we explicitly construct.

#### 1. Introduction

Simple anti-de Sitter hypergravity in three dimensions is a consistent higher spin field theory involving fields of spins 2, 4 and  $\frac{5}{2}$  that is invariant under hypersymmetry, a higher spin fermionic symmetry with spin- $\frac{3}{2}$  parameter. In the limit of zero cosmological constant, the spin-4 field decouples and the theory of the remaining fields reduces to the hypergravity model of [1] that has been recently reformulated as a Chern-Simons theory in [2]. The theory has no local degrees of freedom, but possesses the rich asymptotics of higher spin gauge fields in  $2 + 1$  dimensions described by  $W$ -(super)algebras [3–5], in this case  $W_{(2, \frac{5}{2}, 4)}$  [6].

The asymptotic symmetry algebra has interesting consequences since it implies “hypersymmetry bounds”, in much the same way as supersymmetry implies supersymmetry bounds. In [6] we explicitly derived the hypersymmetry bounds for simple AdS hypergravity and analyzed them for different types of solutions, in particular, for black holes. We showed that the hypersymmetric black holes saturate the bounds and are extremal, in the sense that they lie on the border of the region within which a sensible thermodynamics (real entropy) can be defined.

The purpose of this work is to extend the analysis to hypergravity models with more hypersymmetries (“extended hypergravity”). This is done along the following lines. First, in the next section (section 2), we describe the extended hypergravity models: we discuss the underlying superalgebras and write down the action. Then, in Section 3, we study the asymptotics using well-established methods and show that the asymptotic superalgebra is an extension of the algebra of [6] by  $so(M)$  (respectively  $so(N)$ ) Kac-Moody currents under which the fermionic hypercharges transform in the  $\underline{\mathbf{M}}$  (respectively, in the  $\underline{\mathbf{N}}$ ). We derive the form of the nonlinear hypersymmetry bounds in 4. Next, in Section 5, we construct the black hole solutions and discuss their thermodynamics. Finally, we show that the hypersymmetric black holes are extremal and saturate the hypersymmetry bounds (Section 6). Section 7 collects our concluding remarks.

## 2. Extended anti-de Sitter hypergravities in $2 + 1$ dimensions

$(M, N)$ -extended AdS hypergravities in three-dimensional spacetimes are hypersymmetric extensions of  $sp(4)$  higher spin gravity, described by a Chern-Simons theory with gauge algebra  $sp(4) \oplus sp(4)$  where the gravitational subalgebra  $sp(2) \simeq sl(2, \mathbb{R})$  is principally embedded on each side. The  $sp(4)$  higher spin gravity contains the graviton and a spin-4 field. Here, the word “spin” refers to the conformal weight of the corresponding asymptotic generators in the conformal algebra at infinity, see below.

One may “hypersymmetrize” the  $sp(4)$  higher spin gravity and construct  $(M, N)$ -extended AdS hypergravities, with  $M$  hypersymmetries in one chiral sector and  $N$  hypersymmetries in the other, by embedding  $sp(4) \oplus sp(4)$  in an appropriate superalgebra. We consider hypersymmetric extensions such that the resulting superalgebra has the following properties:

- The bosonic subalgebra is the direct sum  $sp(4) \oplus \mathcal{G}$  where  $\mathcal{G}$  is a compact algebra.
- The fermionic generators transform in the  $\underline{\mathbf{4}}$  of  $sp(4)$  (and in some representation of  $\mathcal{G}$  on which we do not impose any a priori requirement).

The first condition implies that the extra bosonic fields in the theory, coming in addition to the graviton and its spin-4 companion, have all lower spin 1. These extra fields are just the gauge fields associated with the internal  $R$ -symmetry described by  $\mathcal{G}$ . The second condition guarantees that the fermionic fields have all spin  $\frac{5}{2}$  (“hypergravitini”).

The algebra  $sp(4)$  is the algebra underlying anti-de Sitter hypergravity in three dimensions, but it is also the anti-de Sitter algebra in 4 dimensions. As such, its graded extensions have been systematically studied in the early days of supergravity. It turns out that there is only one class of graded extensions, given by  $osp(M|4)$  [7, 8]. Thus, while there are seven distinct types of extended supergravity models

in three dimensions [7–11], there is only one type of extended hypergravity models. The underlying superalgebras are  $osp(M|4) \oplus osp(N|4)$ .

The  $(M, N) = (1, 1)$  case of [6] is described by the superalgebra  $osp(1|4) \oplus osp(1|4)$ . It contains, in addition to the graviton and its spin-4 companion, a spin  $\frac{5}{2}$  field on each chiral side, called the “hypergravitino”. In the extended case, there are more “hypergravitini” and these transform in the  $\underline{\mathbf{M}}$  (respectively, in the  $\underline{\mathbf{N}}$ ) of  $so(M)$  (respectively,  $so(N)$ ). There are also extra gauge fields transforming in the adjoint of  $so(M)$  (respectively,  $so(N)$ ).

For definiteness, we shall focus from now on the chiral sector with superalgebra  $osp(M|4)$ . Similar considerations apply to the other sector. The (anti)commutation relations of  $osp(M|4)$  are explicitly:

$$\begin{aligned}
[L_i, L_j] &= (i - j) L_{i+j}, \\
[L_i, U_m] &= (3i - m) U_{i+m}, \\
[L_i, T^{IJ}] &= 0, \\
[L_i, \mathcal{S}_p^I] &= \left(\frac{3}{2}i - p\right) \mathcal{S}_{i+p}^I, \\
[U_m, U_n] &= \\
&\frac{1}{12} (m - n) \left( (m^2 + n^2 - 4) \left( m^2 + n^2 - \frac{2}{3}mn - 9 \right) - \frac{2}{3} (mn - 6) mn \right) L_{m+n} \\
&\quad + \frac{1}{6} (m - n) (m^2 - mn + n^2 - 7) U_{m+n}, \\
[T^{IJ}, T^{KL}] &= \delta^{IK} T^{JL} - \delta^{IL} T^{JK} - \delta^{JK} T^{IL} + \delta^{JL} T^{IK}, \\
[U_m, T^{IJ}] &= 0, \\
[U_m, \mathcal{S}_p^I] &= \frac{1}{24} (2m^3 - 8m^2p + 20mp^2 + 82p - 23m - 40p^3) \mathcal{S}_{i+p}^I, \\
[T^{IJ}, \mathcal{S}_p^K] &= \delta^{IK} \mathcal{S}_p^J - \delta^{JK} \mathcal{S}_p^I, \\
\{\mathcal{S}_p^I, \mathcal{S}_q^J\} &= \delta^{IJ} \left( U_{p+q} + \frac{1}{12} (6p^2 - 8pq + 6q^2 - 9) L_{p+q} \right) \\
&\quad - \frac{5}{12} (p - q) (2p^2 + 2q^2 - 5) T^{IJ}.
\end{aligned} \tag{1}$$

Here  $L_i$ , with  $i = 0, \pm 1$ , stand for the generators that span the gravitational  $sl(2, \mathbb{R})$  subalgebra, while  $T^{IJ} = -T^{JI}$ , with  $I, J = 1, \dots, M$ , are the spin-0  $so(M)$  generators, which will yield spin-1 fields in the Chern-Simons theory. The  $U_m$  and  $\mathcal{S}_p^I$ , with  $m = 0, \pm 1, \pm 2, \pm 3$  and  $p = \pm \frac{1}{2}, \pm \frac{3}{2}$ , will yield the spin-4 and spin- $\frac{5}{2}$  fields, respectively.

The dynamics of  $(M, N)$ -extended hypergravity follows from the difference of two Chern-Simons actions,  $I = I_{CS}[A^+] - I_{CS}[A^-]$ , with

$$I_{CS}[A] = \frac{k_4}{4\pi} \int str \left[ AdA + \frac{2}{3} A^3 \right], \tag{2}$$

where the level,  $k_4 = k/10$ , is expressed in terms of the Newton constant and the AdS radius according to  $k = \ell/4G$ . In eq. (2)  $str[\dots]$  stands for the supertrace of

the fundamental  $((4 + M) \times (4 + M)$  or  $(4 + N) \times (4 + N))$  matrix representation of  $osp(M|4)$  and the gauge fields  $A^\pm$  correspond to the two independent copies  $osp(M|4)$  and  $osp(N|4)$ . A convenient matrix representation of the generators  $T^{IJ}$  is such that the lower diagonal block is given by

$$(T^{IJ})^K_L = -2\delta^{K[I}\delta^{J]L},$$

and hence

$$str(T^{IJ}T^{KL}) = 4\delta^{K[I}\delta^{J]L}.$$

### 3. Asymptotic structure of extended hypergravities

#### 3.1. Boundary conditions

In order to discuss the boundary conditions, we perform – as it has now become standard – the gauge transformation of [12] that eliminates asymptotically the radial dependence of the connections, so that  $A^\pm = g_\pm^{-1}a^\pm g_\pm + g_\pm^{-1}dg_\pm$ , with

$$a^\pm = a_\varphi^\pm(t, \varphi) d\varphi + a_t^\pm(t, \varphi) dt \quad (3)$$

(to leading order). Then, following the lines of [3–5, 13], we impose that at any fixed time slice  $t = t_0$ , the deviations with respect to the reference background go asymptotically along the lowest (highest)  $sl(2)$ -weight vectors for each  $sl(2)$ -representation occurring in the theory, i.e.,

$$a_\varphi^\pm = L_{\pm 1} - \frac{2\pi}{k}\tilde{\mathcal{L}}^\pm(\varphi)L_{\mp 1} + \frac{\pi}{5k}\mathcal{U}^\pm(\varphi)U_{\mp 3} - \frac{2\pi}{k}\psi_I^\pm(\varphi)\mathcal{S}_{\mp \frac{3}{2}}^I - \frac{5\pi}{k}\mathcal{J}_{IJ}^\pm(\varphi)T^{IJ}. \quad (4)$$

All components  $\mathcal{J}_{IJ}^\pm$  along the internal symmetry generators  $T^{IJ}$ , which are  $sl(2)$ -scalars, are allowed. In (4),  $\tilde{\mathcal{L}}^\pm(\varphi)$  is defined in terms of what will become the Virasoro generators  $\mathcal{L}^\pm$  through

$$\tilde{\mathcal{L}}^\pm = \mathcal{L}^\pm - \frac{5\pi}{2k}\mathcal{J}_{IJ}^\pm\mathcal{J}^{\pm IJ}. \quad (5)$$

The two expressions differ by the familiar Sugawara term quadratic in the currents.

#### 3.2. Asymptotic symmetries

Exactly as in [3–5, 13], one then finds that the fall-off conditions (4) are maintained under a restricted set of gauge transformations,  $\delta a^\pm = d\Omega^\pm + [a^\pm, \Omega^\pm]$ , where, on each slice, the Lie-algebra-valued parameters

$$\Omega^\pm = \Omega^\pm[\epsilon_\pm, \chi_\pm, \zeta_{IJ\pm}, \vartheta_\pm^I], \quad (6)$$

depend on  $(2 + \frac{M^\pm(M^\pm - 1)}{2})$  bosonic and  $M^\pm$  fermionic functions of  $\varphi$ , given by  $\epsilon_\pm, \chi_\pm, \zeta_{IJ\pm}$ , and  $\vartheta_\pm^I$ , respectively. Here, we have set  $M^+ = M$  and  $M^- = N$ . They take the form

$$\begin{aligned} \Omega^\pm[\epsilon_\pm, \chi_\pm, \zeta_{IJ\pm}, \vartheta_\pm^I] &= \epsilon_\pm(\varphi)L_{\pm 1} - \chi_\pm(\varphi)U_{\pm 3} \mp \vartheta_{\pm[I}(\varphi)\mathcal{S}_{\pm \frac{3}{2}}^{I]} \\ &+ \left(\zeta_{IJ\pm}(\varphi) - \frac{5\pi}{k}\epsilon_\pm(\varphi)\mathcal{J}_{IJ}^\pm\right)T^{IJ} + \eta^\pm[\epsilon_\pm, \chi_\pm, \nu_{IJ\pm}, \vartheta_\pm^I], \quad (7) \end{aligned}$$

where the  $\eta^\pm$ 's, and the precise way in which the fields  $\mathcal{L}^\pm, \mathcal{U}^\pm, \mathcal{J}_{IJ}^\pm, \psi^{\pm I}$  transform, are explicitly given in Appendix A. These expressions involve the fields  $\mathcal{L}^\pm, \mathcal{U}^\pm, \mathcal{J}_{IJ}^\pm, \psi^{\pm I}$  and the independent gauge parameters  $\epsilon_\pm, \chi_\pm, \zeta_{IJ\pm}$ , and  $\vartheta_\pm$ , as well as their derivatives with respect to  $\varphi$ .

The boundary conditions (4) define phase space at a given instant of time. Phase space histories fulfill (4) at all times, i.e., take the form (4) with the functions  $\tilde{\mathcal{L}}^\pm, \mathcal{U}^\pm, \psi_I^\pm$  and  $\mathcal{J}_{IJ}^\pm$  now depending also on  $t$ . These boundary conditions are of course preserved by gauge transformations of the form (7) with parameters  $\epsilon_\pm(t, \varphi), \chi_\pm(t, \varphi), \vartheta_{\pm[I]}(t, \varphi), \zeta_{IJ\pm}(t, \varphi)$  that are time-dependent too. In particular, the motion in time is a gauge transformation with gauge parameter  $a_t^\pm$ . This implies that the asymptotic behaviour of  $a_t^\pm$  has to be given by [14, 15]

$$a_t^\pm = \pm \Omega^\pm [\xi_\pm, \mu_\pm, \nu_{IJ\pm}, \varrho_\pm^I] , \quad (8)$$

where  $\Omega^\pm$  is defined through (7), and  $\xi_\pm, \mu_\pm, \nu_{IJ\pm}, \varrho_\pm^I$  can be identified with the “chemical potentials” when one goes to the thermodynamical formulation. Once the temporal components of the vector potential have been chosen, the parameters  $\epsilon_\pm, \chi_\pm, \zeta_{IJ\pm}, \vartheta_\pm^I$  of the residual gauge transformations must fulfill certain differential equations of first order in time expressing that the  $a_t^\pm$ 's are left invariant by the transformations, which may be regarded as “deformed chirality conditions”.

### 3.3. Generators of asymptotic symmetries

Following the canonical approach [16], one finds that the generators of the asymptotic symmetries are

$$\mathcal{Q}^\pm[\epsilon_\pm, \chi_\pm, \vartheta_{I\pm}] = - \int d\varphi (\epsilon_\pm \mathcal{L}^\pm + \chi_\pm \mathcal{U}^\pm - \zeta_{\pm}^{IJ} \mathcal{J}_{IJ}^\pm - i \vartheta_{\pm}^I \psi_I^\pm) . \quad (9)$$

(modulo bulk terms proportional to the constraints that we will not write explicitly and that can be taken strongly equal to zero if one uses the Dirac bracket - which coincides with the Poisson bracket for gauge invariant functions).

Since the Poisson brackets fulfill  $[\mathcal{Q}[\eta_1], \mathcal{Q}[\eta_2]]_{PB} = -\delta_{\eta_1} \mathcal{Q}[\eta_2]$ , the algebra of the canonical generators can be easily found from the transformation law of the fields, and it is explicitly written down in Appendix B.

Expanding in Fourier modes,  $X = \frac{1}{2\pi} \sum_m X_m e^{im\varphi}$ , the asymptotic symmetry algebra reads

$$\begin{aligned} i [L_m, L_n]_{PB} &= (m - n) L_{m+n} + \frac{k}{2} m^3 \delta_{m+n}^0 , \\ i [L_m, U_n]_{PB} &= (3m - n) U_{m+n} , \\ i [L_m, J_n^{IJ}]_{PB} &= -n J_{m+n}^{IJ} , \\ i [L_m, \psi_n^I]_{PB} &= \left( \frac{3}{2} m - n \right) \psi_{m+n}^I , \end{aligned}$$

$$\begin{aligned}
i [U_m, J_n^{IJ}]_{PB} &= 0, \\
i [J_m^{IJ}, (J_{KL})_n]_{PB} &= -4i\delta_{[K}^{[I} (\mathcal{J}_{L]}^J)_{m+n} - \frac{2k}{5} n \delta_K^{[I} \delta_L^{J]} \delta_{m+n}^0, \\
[J_m^{IJ}, \psi_n^K]_{PB} &= 2\delta^{K[I} \psi_{m+n}^{J]} , \\
i [U_m, U_n]_{PB} &= \frac{1}{2^2 3^2} (m-n) (3m^4 - 2m^3 n + 4m^2 n^2 - 2mn^3 + 3n^4) \left( L_{m+n} - \frac{5\pi}{2k} \Lambda_{m+n}^{(2)} \right) \\
&\quad + \frac{1}{6} (m-n) (m^2 - mn + n^2) \mathcal{U}_{m+n} - \frac{2^3 3\pi}{k} (m-n) \Lambda_{m+n}^{(6)} \\
&\quad - \frac{7^2 \pi}{3^2 k} (m-n) (m^2 + 4mn + n^2) \tilde{\Lambda}_{m+n}^{(4)} + \frac{k}{2^3 3^2} m^7 \delta_{m+n}^0, \\
i [U_m, \psi_n^I]_{PB} &= \frac{1}{2^2 3} (m^3 - 4m^2 n + 10mn^2 - 20n^3) \psi_{m+n}^I - \frac{2^3 \pi}{3k} i \Lambda_{m+n}^{(11/2)I} \\
&\quad + \frac{70\pi}{k} i m^2 \Lambda_{m+n}^{(7/2)I} + \frac{\pi}{3k} (23m - 82n) \Lambda_{m+n}^{(9/2)I}, \\
i [\psi_m^I, \psi_n^J]_{PB} &= \delta^{IJ} \left[ U_{m+n} + \frac{1}{2} \left( m^2 - \frac{4}{3} mn + n^2 \right) \left( L_{m+n} - \frac{5\pi}{2k} \Lambda_{m+n}^{(2)} \right) + \frac{k}{6} m^4 \delta_{m+n}^0 \right] \\
&\quad - \frac{5}{6} i (m-n) (m^2 + n^2) J_{m+n}^{IJ} \\
&\quad - \frac{2^5 \pi}{k} (m^2 + n^2) \Lambda_{m+n}^{(2)IJ} + \frac{50\pi}{3k} i \left( m \Lambda_{m+n}^{(3)JI} + n \Lambda_{m+n}^{(3)IJ} \right) - \frac{3\pi}{k} \Lambda_{m+n}^{(4)(IJ)}, \tag{10}
\end{aligned}$$

where  $\Lambda_m^{(l)}$  stand for the Fourier modes of the corresponding nonlinear terms (see Appendix B). The central charge in the Virasoro subalgebra is the same as that for pure gravity [17].

The modes fulfill the following reality conditions:  $(L_m)^* = L_{-m}$ ,  $(U_m)^* = U_{-m}$ ,  $(\psi_m^I)^* = \psi_{-m}^I$ ,  $(\mathcal{J}_m^{IJ})^* = \mathcal{J}_{-m}^{IJ}$  so that the functions  $\mathcal{L}^\pm$ ,  $\mathcal{U}^\pm$ ,  $\psi^\pm$  and  $\mathcal{J}^{IJ}$  are real.

The asymptotic symmetry algebra given above is the classical limit of an extension of the superalgebra  $W_{(2, \frac{5}{2}, 4)}$  of [18, 19]. This extension involves  $M$  spin- $\frac{5}{2}$  fermionic generators  $\psi_m^I$  and  $\frac{M(M-1)}{2}$  Kac-Moody currents  $\mathcal{J}_n^{IJ}$ . The spin- $\frac{5}{2}$  fermionic generators  $\psi_m^I$  are the ‘‘hypersymmetry’’ generators, and transform in the  $\underline{M}$  of  $so(M)$ . The algebra  $W_{(2, \frac{5}{2}, 4)}$  corresponds to  $M = 1$ .

## 4. Hypersymmetry bounds from the asymptotic symmetry algebra

### 4.1. Boundary conditions and spectral flow

We focus for definiteness on the  $+$  copy and drop the subscript ‘‘+’’. Similar considerations apply to the  $-$  sector.

The fermions are subject to boundary conditions of the form

$$\psi_I(\varphi + 2\pi) = R_{IJ} \psi_J(\varphi) \tag{1}$$

where the matrix  $R = (R_{IJ})$  is an element of  $O(M)$ , which we can take to be either the identity, or a fixed element of  $O(M)$  with determinant  $-1$  discussed below. Different boundary conditions are related to these ones by spectral flow [20] (see also [13] for a discussion in the similar AdS<sub>3</sub> extended supergravity context).

When  $M$  is odd, one may assume  $R = \mathbb{1}$  (periodic boundary conditions) or  $R = -\mathbb{1}$  (antiperiodic boundary conditions). In both cases, the affine generators  $\mathcal{J}_{IJ}$  are periodic and the corresponding affine algebra is untwisted. When  $M$  is even,  $M = 2r$ , one may assume  $R = \mathbb{1}$  (periodic boundary conditions) or, if  $R \neq \mathbb{1}$ , that it defines an outer automorphism of  $SO(2r)$ . In that latter case, the affine generators are not periodic and the affine algebra is twisted.

We shall restrict the analysis to periodic boundary conditions (“Ramond case”). This is motivated by the fact that we are interested in black holes. The situations found in (1,1) hypergravity and supergravity indicate that black hole solutions naturally admit in both cases the periodic spin structure [6, 21]. Note that the antiperiodic case (“Neveu-Schwarz” case) is automatically included when  $M$  is even since then, as mentioned above, it can be related to the periodic case by spectral flow [20] ( $-\mathbb{1} \in SO(2m)$ ). With periodic boundary conditions, the Fourier labels  $m$  in  $X = \frac{1}{2\pi} \sum_m X_m e^{im\varphi}$  are integers for all fields  $X$ .

#### 4.2. Hypersymmetry bounds

The Poisson Bracket of the fermionic generator of the asymptotic symmetry hyperalgebra in (10), implies interesting hypersymmetry bounds. These were discussed in great generality in [6]. Here, we focus on the bounds that hold in the context of periodic boundary conditions.

We consider bosonic configurations carrying global charges with only zero modes, given by  $L_0 = 2\pi\mathcal{L}$ ,  $U_0 = 2\pi\mathcal{U}$  and  $J_0^{IJ} = 2\pi\mathcal{J}^{IJ}$ , for each copy. Furthermore, we assume without loss of generality that the affine Kac-Moody currents have been brought to the Cartan subalgebra by conjugation, so that  $\mathcal{J}^{IJ}$  has only non-vanishing components  $\mathcal{J}^{2i-1\,2i}$  ( $i = 1, 2, \dots$ ,  $\text{rank}SO(M) = [\frac{M}{2}] = r$ ). We set  $M = 2r$  when  $M$  is even, or  $M = 2r + 1$  when  $M$  is odd, and

$$\mathcal{J}^{2i-1\,2i} = j_i \epsilon^{2i-1\,2i} \quad (2)$$

The anticommutators of the hypersymmetry generators with  $m = -n = p \geq 0$  are then found to reduce to

$$(2\pi)^{-1} \left( \hat{\psi}_p^I \hat{\psi}_{-p}^J + \hat{\psi}_{-p}^J \hat{\psi}_p^I \right) = B_p^{IJ}, \quad (3)$$

with

$$\begin{aligned} B_p^{IJ} = & \left( \mathcal{U} + \frac{3\pi}{k} \tilde{\mathcal{L}}^2 \right) \delta^{IJ} + \frac{500\pi^2}{3k^2} \mathcal{J}_K^{IJ} \left( \tilde{\mathcal{L}} \mathcal{J}^{JK} + \frac{5\pi}{k} \mathcal{J}_{MN} \mathcal{J}^{JM} \mathcal{J}^{KN} \right) \\ & - \frac{100i\pi}{3k} \left( \tilde{\mathcal{L}} \mathcal{J}^{IJ} + \frac{10\pi}{k} \mathcal{J}_M^I \mathcal{J}^{ML} \mathcal{J}_L^J \right) p + \frac{5}{3} \left( \delta^{IJ} \tilde{\mathcal{L}} + \frac{30\pi}{k} \mathcal{J}_K^I \mathcal{J}^{JK} \right) p^2 \\ & - \frac{10}{3} i \mathcal{J}^{IJ} p^3 + \frac{k}{12\pi} \delta^{IJ} p^4. \end{aligned} \quad (4)$$

Note that

$$(B_p^{IJ})^\dagger = B_p^{JI}, \quad (5)$$

and as one sees explicitly from (4),  $B_p^{IJ}$  has both a real symmetric part and a pure imaginary, antisymmetric part.

Now, the hermitian operator  $\hat{\psi}_p^I \hat{\psi}_{-p}^I + \hat{\psi}_{-p}^I \hat{\psi}_p^I$  is positive definite for each  $I$  and  $p$ . This implies, in the classical limit, that the global charges fulfill the bounds

$$B_p^{II} \geq 0, \quad (6)$$

(no summation over  $I$ ). The bound  $B_0^{II} \geq 0$  for  $p = 0$  reads

$$B_0^{II} \equiv \left( \mathcal{U} + \frac{3\pi}{k} \tilde{\mathcal{L}}^2 \right) + \frac{500\pi^2}{3k^2} \left( \tilde{\mathcal{L}}(j_i)^2 + \frac{5\pi}{k} (j_i)^4 \right) \geq 0, \quad (7)$$

with  $I = 1, 2, \dots, 2r$ . Note that when  $M$  is odd, there is an additional bound corresponding to  $I = 2r + 1$ ,

$$B_0^{2r+1, 2r+1} \equiv \left( \mathcal{U} + \frac{3\pi}{k} \tilde{\mathcal{L}}^2 \right) \geq 0. \quad (8)$$

These bounds are manifestly nonlinear.

One can express the bounds for  $p > 0$  in terms of the bounds for  $p = 0$  as

$$B_p^{II} = B_0^{II} + \frac{5}{3} \left( \tilde{\mathcal{L}} + \frac{30\pi}{k} (j_i)^2 \right) p^2 + \frac{k}{12\pi} p^4 \geq 0.$$

Now, in the black hole case, one must have  $\tilde{\mathcal{L}} \geq 0$  (see below) and so one finds that the bounds with  $p > 0$  are automatic consequences of bounds with  $p = 0$ , which are thus the strongest.

One can derive further bounds involving the mixed terms  $B_p^{IJ}$  with  $J \neq I$ . To illustrate the procedure, consider for definiteness  $I = 1$  and  $J = 2$ , for which  $B_p^{12}$  does not identically vanish. Form the complex fields  $\chi_p = \psi_p^1 + i\psi_p^2$  and  $\omega_p = \psi_p^1 - i\psi_p^2$ . From the conditions  $\chi_p(\chi_p)^\dagger + (\chi_p)^\dagger \chi_p \geq 0$  and  $\omega_p(\omega_p)^\dagger + (\omega_p)^\dagger \omega_p \geq 0$ , one gets

$$B_p^{11} + B_p^{22} \geq \pm i(B_p^{12} - B_p^{21}), \quad (9)$$

i.e., given that  $B_p^{11} = B_p^{22}$  and  $B_p^{12} = -B_p^{21}$ ,

$$B_p^{11} \geq \pm i B_p^{12}. \quad (10)$$

In general, this bound is independent from the previous ones, but it is not so in the black hole case. Indeed, the condition (10) can be conveniently factorized as

$$\left[ \left( p - \frac{10\pi}{k} j_1 \right)^2 + \lambda_{[+]}^2 \right] \left[ \left( p - \frac{10\pi}{k} j_1 \right)^2 + \lambda_{[-]}^2 \right] \geq 0, \quad (11)$$

where  $\pm \lambda_{[+]}$  and  $\pm \lambda_{[-]}$  correspond to the eigenvalues of the  $sp(4)$  dynamical gauge fields introduced in the next section. In the black hole case, these eigenvalues are necessarily real (see below), so that for this class of solutions the bounds in (11) are clearly fulfilled. One refers for this reason to the bounds (7) with  $p = 0$  as the “strongest bounds” in the black hole context.



## 5. Black holes

### 5.1. Black hole connection and regularity conditions

Higher spin black holes generalizing the pure gravity black hole [22, 23] have been investigated first in the pioneering work [24–26], reviewed in [27]. A different class of black hole solutions differing in their asymptotics was subsequently derived in [14, 15, 28]. We follow this approach as it is clearly compatible with the asymptotic  $W$ -symmetry algebra exhibited above.

In the absence of a well-defined spacetime geometry, higher spin black holes are defined through the Euclidean continuation [24, 25], as regular flat connections on the solid torus with well-defined thermodynamics (real entropy). We follow this point of view but, however, as in [15], we impose the above boundary conditions on the connection and not ones that would modify the asymptotic behaviour of  $a_\varphi^{pm}$ . For the  $(M, M)$ -extended AdS hypergravity theory, the Euclidean connection that describes the black holes is a direct generalization of the simple hypergravity black hole of [6] and can be written as

$$a = \left( L_1 - \frac{2\pi}{k} \tilde{\mathcal{L}} L_{-1} + \frac{\pi}{5k} \mathcal{U} U_{-3} - \frac{5\pi}{k} \mathcal{J}_{IJ} T^{IJ} \right) d\varphi - \left\{ i\xi \left( L_1 - \frac{2\pi}{k} \tilde{\mathcal{L}} L_{-1} + \frac{\pi}{5k} \mathcal{U} U_{-3} + \frac{5\pi}{k} \mathcal{J}_{IJ} T^{IJ} \right) - i\mu \left[ U_3 - \frac{6\pi}{k} \tilde{\mathcal{L}} U_1 - \frac{22}{15} \frac{\pi^2}{k^2} \left( \mathcal{U} + \frac{60\pi}{11k} \tilde{\mathcal{L}}^2 \right) \tilde{\mathcal{L}} U_{-3} + \frac{6\pi}{k} \mathcal{U} L_{-1} + \frac{\pi}{k} \left( \mathcal{U} + \frac{12\pi}{k} \tilde{\mathcal{L}}^2 \right) U_{-1} \right] - i\nu_{IJ} T^{IJ} \right\} d\tau, \quad (12)$$

where  $\tilde{\mathcal{L}}$  is given by

$$\tilde{\mathcal{L}} = \mathcal{L} - \frac{5\pi}{2k} \mathcal{J}_{IJ} \mathcal{J}^{IJ}, \quad (13)$$

while the components of the zero modes of the  $so(M)$  Kac-Moody currents, and their corresponding chemical potentials are constrained to commute by the field equations, and hence

$$\mathcal{J}_{[I}^K \nu_{J]K} = 0. \quad (14)$$

As above, if we assume that  $\mathcal{J}_{IJ} T^{IJ}$  belong to the Cartan subalgebra of  $so(M)$ , this condition implies that the chemical potentials  $\nu_{IJ} T^{IJ}$  also do. Therefore,

$$\nu_{2i-1\ 2i} = \nu_i \epsilon_{2i-1\ 2i}.$$

Due to the fact that the thermal cycles are contractible, the holonomy of the gauge fields along them has to be trivial. These are the so-called “regularity conditions”. For the branch of solutions that is continuously connected to the BTZ black hole [22, 23], possibly endowed with an  $so(M)$  field, the regularity conditions read

$$e^{a_\tau^{sp(4)}} \cdot e^{i(\nu_{IJ} + \frac{5\pi}{k} \xi \mathcal{J}_{IJ}) T^{IJ}} = \begin{pmatrix} -\mathbb{1}_{4 \times 4} & 0 \\ 0 & \mathbb{1}_{M \times M} \end{pmatrix}. \quad (15)$$

Hence, the chemical potentials fulfill

$$\nu_i + \frac{5\pi}{k} \xi j_i = 2\pi n_i, \quad (16)$$

where  $n_i$  stands for a set of integers, and

$$\xi = \frac{\pi}{5^2 2} \left[ \frac{3^3 \lambda_{[-]}^3 - 41 (3\lambda_{[+]} - \lambda_{[-]}) \lambda_{[-]} \lambda_{[+]} - 3^2 \lambda_{[+]}^3}{(\lambda_{[-]}^2 - \lambda_{[+]}^2) \lambda_{[-]} \lambda_{[+]}} \right], \quad (17)$$

$$\mu = \frac{3\pi}{5} \left[ \frac{3\lambda_{[-]} - \lambda_{[+]}}{(\lambda_{[-]}^2 - \lambda_{[+]}^2) \lambda_{[-]} \lambda_{[+]}} \right], \quad (18)$$

with  $\lambda_{[\pm]}$  given by

$$\lambda_{[\pm]}^2 = \frac{10\pi}{k} \left( \tilde{\mathcal{L}} \pm \frac{4}{5} \sqrt{\tilde{\mathcal{L}}^2 - \frac{3k}{16\pi} \mathcal{U}} \right). \quad (19)$$

One gets exactly the same regularity condition in the  $sp(4)$  sector (in terms of  $\tilde{\mathcal{L}}$ ) as in the simple hypergravity case considered in [6] (or, for that matter, as in the case of pure  $sp(4)$  gravity). We also note that the natural value for the integers  $n_i$  characterizing the holonomy of the internal  $SO(M)$  symmetry is  $n_i = 0$  since otherwise there might appear to be a  $\delta$ -function source of quantized strength in the non-gravitational, internal, sector, but we shall temporarily allow for more general  $n_i$ 's to see how these integers enter the entropy. One could similarly allow for more general solutions of the regularity conditions involving different integers in the  $sp(4)$  sector; the above choice corresponds to the BTZ branch.

## 5.2. Entropy

We use the correct canonical expression for the black hole entropy adapted to the above boundary conditions derived first in [29, 30], which can also be alternatively written according to [15, 31, 32] as

$$S = -2k_4 \text{Im}(\text{str}[a_\tau a_\varphi]) \quad (20)$$

(in the conventions of [15]). Once evaluated for the solution in (12), the black hole entropy becomes

$$S = 8\pi \text{Re} \left[ \xi \mathcal{L} + 2\mu \mathcal{U} + \frac{1}{2} \nu^{IJ} \mathcal{J}_{IJ} \right]. \quad (21)$$

Plugging then the expressions for the chemical potentials into (21) allows one to express the black hole entropy in terms of the (extensive) global charges. One gets

$$S = \frac{2\pi k}{5} \text{Re} \left( 3\lambda_{[+]} + \lambda_{[-]} + \frac{5}{2k} n_i j_i \right). \quad (22)$$

For the natural  $so(M)$  holonomy  $n_i = 0$ , this expression becomes

$$S = \frac{2\pi k}{5} \text{Re} (3\lambda_{[+]} + \lambda_{[-]}) . \quad (23)$$

It is only for this branch that the black hole entropy reduces to the horizon area over  $4G$  when the spin-4 field is turned off.

Two points are worth being pointed out: (i) as can already be seen for the coupled pure gauge-gravitational fields without higher spin gauge fields described by the gauge algebra  $sl(2, \mathbb{R}) \oplus so(M)$ , the  $so(M)$  gauge fields are “gravitationally stealth” in the sense of [33], i.e., they do not generate a back reaction on the metric because their contribution to the stress energy vanishes; they only contribute to a redefinition of the asymptotic Virasoro generators; (ii) the black hole entropy of the  $n_i = 0$  branch (23) is also blind to them if one expresses it in terms of the tilted Virasoro generators but not so if one uses the Virasoro generators fulfilling the above asymptotic algebra and directly related to the mass  $M$  and the angular momentum  $J$ . The black hole entropy can also detect non-vanishing  $n_i$ , see (22).

For the  $n_i = 0$  branch that we consider from now on, the Lorentzian continuation of the entropy reads

$$S = \pi \sqrt{\frac{2}{5}} \pi k \left[ \sqrt{\tilde{\mathcal{L}}^+} \left( \sqrt{1 - \frac{4}{5}} \sqrt{1 - \frac{3k\mathcal{U}^+}{16\pi(\tilde{\mathcal{L}}^+)^2}} + 3 \sqrt{1 + \frac{4}{5}} \sqrt{1 - \frac{3k\mathcal{U}^+}{16\pi(\tilde{\mathcal{L}}^+)^2}} \right) + \sqrt{\tilde{\mathcal{L}}^-} \left( \sqrt{1 - \frac{4}{5}} \sqrt{1 - \frac{3k\mathcal{U}^-}{16\pi(\tilde{\mathcal{L}}^-)^2}} + 3 \sqrt{1 + \frac{4}{5}} \sqrt{1 - \frac{3k\mathcal{U}^-}{16\pi(\tilde{\mathcal{L}}^-)^2}} \right) \right]. \quad (24)$$

Requiring the entropy to be well-defined, i.e., being real and positive, implies that the eigenvalues  $\lambda_{[\pm]}$  should be real. This forces then the spin-4 charges to be bounded according to

$$-(\tilde{\mathcal{L}}^\pm)^2 \leq \frac{k}{3\pi} \mathcal{U}^\pm \leq \frac{2^4}{3^2} (\tilde{\mathcal{L}}^\pm)^2, \quad (25)$$

in addition to  $\tilde{\mathcal{L}}^\pm \geq 0$ . The bounds are saturated in the extremal cases, and only the lower one in (25) corresponds to the hypersymmetry bound aforementioned. Note that the range of positive spin-4 charges is larger than that of the negative ones.

## 6. Killing vector-spinors

Bosonic configurations that admit unbroken hypersymmetries have to fulfill the following Killing vector-spinor equation

$$\delta a = d\theta + [a, \theta] = 0, \quad (26)$$

where the parameter  $\theta$  is purely fermionic, given by  $\theta = \theta_I^p \mathcal{S}_p^I$  for both copies, and globally well-defined.

Equivalently, the Killing vector-spinor equation can be obtained from promoting the corresponding asymptotic symmetries to hold everywhere and not just asymptotically. Therefore, in the case of the plus copy ( $a_\varphi = a_\varphi^+$ ), the fermionic parameter

is of the form  $\theta = \Omega^+ [0, 0, 0, \vartheta^I]$ , which explicitly reads

$$\begin{aligned} \theta = & -\vartheta_I \mathcal{S}_{\frac{3}{2}}^I + \left( \vartheta'_I + \frac{10}{k} \mathcal{J}_I^K \vartheta_K \right) \mathcal{S}_{\frac{1}{2}}^I - \frac{1}{2} \left( \vartheta''_I - \frac{6\pi}{k} \tilde{\mathcal{L}} \vartheta_I - \frac{100\pi^2}{k^2} \mathcal{J}_{IK} \mathcal{J}^{JK} \vartheta_J \right. \\ & + \frac{20\pi}{k} \mathcal{J}_I^K \vartheta'_K \left. \right) \mathcal{S}_{-\frac{1}{2}}^I + \frac{1}{6} \left[ \left( \vartheta''_I + \frac{30\pi}{k} \mathcal{J}_I^K \vartheta'_K - \frac{14\pi}{3k} \tilde{\mathcal{L}} \vartheta_I - \frac{300\pi^2}{k^2} \mathcal{J}_{IK} \mathcal{J}^{JK} \vartheta_J \right)' \right. \\ & \left. - \frac{140\pi^2}{k^2} \left( \tilde{\mathcal{L}} \mathcal{J}_I^J + \frac{50\pi}{7k} \mathcal{J}_{IM} \mathcal{J}^{MK} \mathcal{J}_K^J \right) \vartheta_J \right] \mathcal{S}_{-\frac{3}{2}}^I. \end{aligned} \quad (27)$$

The condition that  $a_\varphi$  should be left strictly unchanged then implies that the parameters  $\vartheta_I$  should satisfy the following differential equations:

$$\begin{aligned} & \left[ \left( \mathcal{U} + \frac{3\pi}{k} \tilde{\mathcal{L}}^2 \right) \delta_I^J + \frac{500\pi^2}{3k^2} \mathcal{J}_{IK} \left( \tilde{\mathcal{L}} \mathcal{J}^{JK} + \frac{5\pi}{k} \mathcal{J}_{MN} \mathcal{J}^{JM} \mathcal{J}^{KN} \right) \right] \vartheta_J \\ & - \frac{100\pi}{3k} \left( \mathcal{J}_I^K \tilde{\mathcal{L}} + \frac{10\pi}{k} \mathcal{J}_{IM} \mathcal{J}^{ML} \mathcal{J}_L^K \right) \vartheta'_K \\ & - \frac{5}{3} \left( \delta_I^K \tilde{\mathcal{L}} + \frac{30\pi}{k} \mathcal{J}_{IJ} \mathcal{J}^{KJ} \right) \vartheta''_K + \frac{10}{3} \mathcal{J}_I^K \vartheta'''_K + \frac{k}{12\pi} \vartheta''''_I = 0. \end{aligned} \quad (28)$$

From the experience gathered with black holes within (1,1) hypergravity or supergravity [6, 21], it is reasonable to assume that the fermionic parameters are constant, given by  $\vartheta^I = \vartheta_0^I$ . The Killing vector-spinor equations (28) then reduce to

$$\left[ \left( \mathcal{U} + \frac{3\pi}{k} \tilde{\mathcal{L}}^2 \right) \delta_I^J + \frac{500\pi^2}{3k^2} \mathcal{J}_{IK} \left( \tilde{\mathcal{L}} \mathcal{J}^{JK} + \frac{5\pi}{k} \mathcal{J}_{MN} \mathcal{J}^{JM} \mathcal{J}^{KN} \right) \right] \vartheta_J = B_0^{IK} \vartheta_K = 0, \quad (29)$$

which clearly admit non trivial solutions if the matrix  $B_0^{IK}$  has zero eigenvalues. Since black holes are well-defined provided  $\tilde{\mathcal{L}} \geq 0$ , and the spin-4 charges fulfill eq. (25), the Killing vector-spinor equations (28) possess non trivial solutions only when the lower bound in (25) is saturated, i.e., only for negative spin-4 charges given by

$$\mathcal{U} = -\frac{3\pi}{k} (\tilde{\mathcal{L}})^2.$$

When this condition is fulfilled,

- there is at least one Killing vector-spinor when  $M$  is odd (corresponding to  $B_0^{2r+1, 2r+1} = 0$ ) and more if some currents  $j_i$  vanish;
- there are Killing vector-spinors when  $M$  is even only if some currents  $j_i$  vanish.
- The maximum number of hypersymmetries is thus  $M$ . It is attained when all the  $j_i$ 's vanish and correspond to  $\frac{M}{4}$ -hypersymmetry, in agreement with the  $\frac{1}{4}$ -hypersymmetry found in [6] for  $M = 1$ .

It is straightforward to verify that the remaining Killing vector-spinor equation, that come from preserving the form of the Lagrange multiplier  $a_t$  globally, is also fulfilled.

## 7. Conclusions

In this note, we have extended the analysis of hypersymmetry bounds of [6] to extended  $\text{AdS}_3$  hypergravity. These bounds follow from the asymptotic symmetry superalgebra and involve the charges nonlinearly. Just as in [6], we have found that the bounds are saturated by a class of extremal black holes, which are hypersymmetric (i.e., possess Killing vector-spinors). However, not all extremal black holes are hypersymmetric. The fact that extremality and super/hypersymmetry do not coincide in the context of higher spin black holes has been discussed recently in the thorough work [34], which focuses on (an appropriate real form of) the superalgebra  $sl(3|2)$ .

Hypersymmetric solutions of a different types (solitons) have been also explored in [6]. The extension of that analysis to extended hypergravity is left for future study.

Finally, we note that nonlinear bounds have also been found in the context of asymptotically flat solutions of hypergravity in the case of fermionic fields of spin  $s = n + \frac{3}{2}$ , with  $n > 0$  (which, in the case of  $n = 0$ , i.e., supergravity, turn out to be linear) [35].

## 8. Acknowledgments

D.T. holds a Marina Solvay fellowship. The work of M.H. and D.T. is partially supported by the ERC through the “SyDuGraM” Advanced Grant, by FNRS-Belgium (convention FRFC PDR T.1025.14 and convention IISN 4.4503.15) and by the “Communaut Franaise de Belgique” through the ARC program. The work of A.P., D.T. and R.T. is partially funded by the Fondecyt grants N° 11130262, 11130260, 1130658, 1121031. The Centro de Estudios Científicos (CECs) is funded by the Chilean Government through the Centers of Excellence Base Financing Program of Conicyt.

## Appendix A. Explicit form of the asymptotic symmetries

The Lie-algebra-valued parameter  $\eta^\pm$  that appears in the asymptotic gauge symmetries spanned by  $\Omega^\pm$  in eq. (6) is given by

$$\begin{aligned}
\eta^\pm [\epsilon_\pm, \chi_\pm, \vartheta_\pm^I] = & -\frac{3\pi}{k} \left( i\psi_I^\pm \vartheta_\pm^I + \frac{2}{3} \epsilon_\pm \tilde{\mathcal{L}}^\pm + 2\chi_\pm \mathcal{U}^\pm - \frac{k}{6\pi} \epsilon_\pm'' \right) L_{\mp 1} \mp \epsilon_\pm' L_0 \\
& + \frac{6\pi}{k} \left( \chi_\pm \tilde{\mathcal{L}}^\pm - \frac{k}{12\pi} \chi_\pm'' \right) U_{\pm 1} \mp \frac{2\pi}{k} \left( \chi_\pm \tilde{\mathcal{L}}^{\pm'} + \frac{8}{3} \chi_\pm' \tilde{\mathcal{L}}^\pm - \frac{k}{12\pi} \chi_\pm''' \right) U_0 \\
& - \frac{\pi}{2k} \left[ i\psi_I^\pm \vartheta_\pm^I + 2 \left( \mathcal{U}^\pm - \frac{1}{2} \tilde{\mathcal{L}}^{\pm''} + \frac{12\pi}{k} (\tilde{\mathcal{L}}^\pm)^2 \right) \chi - \frac{11}{3} \chi_\pm' \tilde{\mathcal{L}}^{\pm'} \right. \\
& - \frac{14}{3} \chi_\pm'' \tilde{\mathcal{L}}^\pm + \frac{k}{12\pi} \chi_\pm^{(4)} \left. \right] U_{\mp 1} \pm \chi_\pm' U_{\pm 2} \pm \frac{\pi}{2k} \left[ i\psi_I^\pm \vartheta_\pm^{I'} + \frac{1}{5} i\psi_I^{\pm'} \vartheta_\pm^I \right. \\
& + \frac{8\pi}{k} i\mathcal{J}_J^{I\pm} \psi_I^\pm \vartheta_\pm^{J'} - \frac{5}{3} \chi_\pm'' \tilde{\mathcal{L}}^{\pm'} - \frac{4}{3} \tilde{\mathcal{L}}^\pm \chi_\pm''' + \frac{2}{5} \left( \mathcal{U}^\pm - \frac{1}{2} \tilde{\mathcal{L}}^{\pm''} + \frac{18\pi}{k} (\tilde{\mathcal{L}}^\pm)^2 \right)' \chi_\pm \\
& + \frac{6}{5} \left( \mathcal{U}^\pm - \frac{7}{9} \tilde{\mathcal{L}}^{\pm''} + \frac{44\pi}{3k} (\tilde{\mathcal{L}}^\pm)^2 \right) \chi_\pm' + \frac{k}{60\pi} \chi_\pm^{(5)} \left. \right] U_{\mp 2} \\
& - \frac{\pi}{4k} \left\{ i\psi_I^\pm \vartheta_\pm^{I''} + \frac{1}{15} i \left( \psi_I^{\pm''} - 2^4 \frac{5\pi}{k} \tilde{\mathcal{L}}^\pm \psi_I^\pm + 2^3 \frac{5\pi}{k} \mathcal{J}_I^{J\pm} \psi_J^{\pm'} + 2^2 \frac{35\pi}{k} \mathcal{J}_I^{J\pm'} \psi_J^\pm \right. \right. \\
& - \frac{10^3 \pi^2}{k^2} \mathcal{J}_K^{J\pm} \mathcal{J}_I^{K\pm} \psi_J^\pm \left. \right) \vartheta_\pm^I + \frac{2}{5} i \left( \psi_I^{\pm'} + \frac{40\pi}{k} \mathcal{J}_I^{J\pm} \psi_J^{\pm'} \right) \vartheta_\pm^{I'} - \chi_\pm''' \tilde{\mathcal{L}}^{\pm'} - \frac{4}{5} \epsilon_\pm \mathcal{U}^\pm \\
& + \frac{2}{3} \left( \mathcal{U}^\pm - \frac{13}{10} \mathcal{L}^{\pm''} + \frac{272\pi}{15k} (\tilde{\mathcal{L}}^\pm)^2 \right) \chi_\pm'' + \frac{8}{15} \left( \mathcal{U}^\pm - \frac{17}{24} \tilde{\mathcal{L}}^{\pm''} + \frac{241\pi}{12k} (\tilde{\mathcal{L}}^\pm)^2 \right)' \chi_\pm' \\
& + \frac{40\pi}{3k} \left[ i\psi_I^\pm \psi^{\pm I'} - \frac{12\pi}{5k} (\tilde{\mathcal{L}}^\pm)^3 - \frac{11}{5^2} \mathcal{U}^\pm \tilde{\mathcal{L}}^\pm + \frac{3^2}{5^2} (\tilde{\mathcal{L}}^{\pm'})^2 + \frac{23}{50} \tilde{\mathcal{L}}^{\pm''} \tilde{\mathcal{L}}^\pm \right. \\
& + \frac{k}{10^2 \pi} \left( \mathcal{U}^\pm - \frac{1}{2} \tilde{\mathcal{L}}^{\pm''} \right)'' + \frac{10\pi}{k} i\mathcal{J}^{IJ\pm} \psi_I^\pm \psi_J^\pm \left. \right] \chi_\pm - \frac{5}{9} \chi_\pm^{(4)} \mathcal{L}^\pm + \frac{k}{180\pi} \chi_\pm^{(6)} \left. \right\} U_{\mp 3} \\
& - \frac{2\pi}{k} \left\{ \epsilon_\pm \psi_I^\pm + \frac{1}{2} \vartheta_{\pm I} \mathcal{L}^{\pm'} + \frac{7}{6} \left[ \delta_I^J \mathcal{L}^\pm - \frac{15}{7} \left( \mathcal{J}_I^{J\pm'} - \frac{10}{k} \mathcal{J}^{JK\pm} \mathcal{J}_{IK}^\pm \right) \right] \vartheta_{\pm J} \right. \\
& - \frac{5}{2} \mathcal{J}_I^{J\pm} \vartheta_{\pm J}'' - \frac{5}{3} \left[ \psi_I^{\pm''} - \frac{52\pi}{5k} \left( \tilde{\mathcal{L}}^\pm \psi_I^\pm - \frac{25}{13} \mathcal{J}_I^{J\pm} \psi_J^{\pm'} - \frac{25}{26} \mathcal{J}_I^{J\pm'} \psi_J^\pm \right. \right. \\
& + \frac{125}{13} \psi_P^\pm \mathcal{J}^{KP\pm} \mathcal{J}_{KI}^\pm \left. \right) \left. \right] \chi_\pm - \frac{25}{6} \left( \psi_I^{\pm'} + \frac{10\pi}{k} \psi_J^\pm \mathcal{J}_I^{J\pm} \right) \chi_\pm' \\
& - \frac{5}{6} \left[ \mathcal{J}_{IJ}^{\pm''} - \frac{10\pi}{k} \left( \mathcal{J}_I^{K\pm'} \mathcal{J}_{JK}^\pm + \frac{7}{5} \tilde{\mathcal{L}} \mathcal{J}_{IJ}^\pm + 2 \mathcal{J}_I^{K\pm} \mathcal{J}_{JK}^{\pm'} + \frac{10\pi}{k} \mathcal{J}^{MN\pm} \mathcal{J}_{IM}^\pm \mathcal{J}_{JN}^\pm \right) \right] \vartheta_\pm^J \\
& - \frac{17}{6} \chi_\pm'' \psi_I^\pm - \frac{k}{12\pi} \vartheta_{\pm I}''' \left. \right\} \mathcal{S}_{\mp \frac{3}{2}}^I \pm \frac{3\pi}{k} \left[ \left( \delta_I^J \tilde{\mathcal{L}}^\pm - \frac{5}{3} \mathcal{J}_I^{J\pm'} + \frac{50}{3k} \mathcal{J}^{JK\pm} \mathcal{J}_{IK}^\pm \right) \vartheta_{\pm J} \right. \\
& - 5 \chi_\pm' \psi_I^\pm - \frac{10}{3} \mathcal{J}_I^{J\pm} \vartheta_{\pm J}' - \frac{10}{3} \left( \psi_I^{\pm'} + \frac{10\pi}{k} \psi_J^\pm \mathcal{J}_I^{J\pm} \right) \chi_\pm - \frac{k}{6\pi} \vartheta_{\pm I}'' \left. \right] \mathcal{S}_{\mp \frac{1}{2}}^I \\
& + \frac{20\pi}{k} \left( \chi_\pm \psi_I^\pm + \frac{1}{2} \mathcal{J}_{IJ}^\pm \vartheta_\pm^J + \frac{k}{20\pi} \vartheta_{\pm I}' \right) \mathcal{S}_{\pm \frac{1}{2}}^I,
\end{aligned}$$

while the transformation laws of the fields  $\mathcal{L}^\pm$ ,  $\mathcal{U}^\pm$ ,  $\mathcal{J}_{IJ}^\pm$ ,  $\psi^\pm$ , explicitly reads

$$\begin{aligned}\delta\mathcal{L}^\pm &= 2\epsilon'_\pm\mathcal{L}^\pm + \epsilon_\pm\mathcal{L}^{\pm'} - \frac{k}{4\pi}\epsilon''''_\pm + 3\mathcal{U}^{\pm'}\chi_\pm + 4\mathcal{U}^\pm\chi'_\pm + \frac{5}{2}i\psi_I^\pm(\vartheta_\pm^I)' + \frac{3}{2}i\psi_I^{\pm'}\vartheta_\pm^I - \mathcal{J}_{IJ}^\pm(\zeta_\pm^{IJ})', \\ \delta j_{IJ}^\pm &= \epsilon'_\pm j_{IJ}^\pm + \epsilon_\pm j_{IJ}^{\pm'} + 4\mathcal{J}_{[I}^{\pm K}\zeta_{J]K}^\pm + \frac{k}{5\pi}\zeta_{IJ}^{\pm'} - 2i\psi_{[I}^\pm\vartheta_{\pm|J]}',\end{aligned}$$

$$\begin{aligned}\delta\psi_I^\pm &= \frac{5}{2}\epsilon'_\pm\psi_I^\pm + \epsilon_\pm\psi_I^{\pm'} + 2\zeta_I^K\psi_K^\pm - \left[\left(\mathcal{U}^\pm - \frac{1}{2}\tilde{\mathcal{L}}^{\pm''}\right)\delta_I^K + \frac{5}{6}(\mathcal{J}_I^{\pm K})'''' + \frac{3\pi}{k}\Lambda_{\pm I}^{(4)K}\right]\vartheta_{\pm K}^I \\ &\quad + \frac{5}{3}\left[\left(\tilde{\mathcal{L}}^\pm\delta_I^K - 2(\mathcal{J}_I^{\pm K})'\right)\vartheta_{\pm K}' - \frac{k}{20\pi}\vartheta_{\pm I}''''\right]' + \frac{82\pi}{3k}\left(\Lambda_\pm^{(9/2)'} - \frac{23}{82}\Lambda_\pm^{(11/2)} - \frac{5k}{82\pi}\psi_I^{\pm''''}\right)\chi_\pm \\ &\quad - 7\left(\psi_I^{\pm'} + \frac{10\pi}{k}\Lambda_{\pm I}^{(7/2)}\right)\chi_\pm'' + \frac{35\pi}{k}\left(\Lambda_{\pm I}^{(9/2)} - \frac{k}{6\pi}\psi_I^{\pm''}\right)\chi_\pm' - \frac{35}{12}\chi_\pm'''\psi_I^\pm \\ &\quad - \frac{5}{3}\left[(\mathcal{J}_I^{\pm K})' + \frac{30\pi}{k}\Lambda_{\pm I}^{(2)K}\right]\vartheta_{\pm K}'' - \frac{10}{3}\mathcal{J}_{\pm I}^K\vartheta_{\pm K}'''' + \frac{10^2}{3k}\Lambda_{\pm I}^{(3)K}\vartheta_{\pm K}',\end{aligned}$$

$$\begin{aligned}\delta\mathcal{U}^\pm &= 4\epsilon'_\pm\mathcal{U}^\pm + \epsilon_\pm\mathcal{U}^{\pm'} + \frac{23\pi}{3k}i\left(\Lambda_{\pm I}^{(11/2)} + \Lambda_{\pm I}^{(9/2)'} + \frac{210}{23}\Lambda_{\pm I}^{(7/2)''} - \frac{k}{92\pi}\psi_I^{\pm''''}\right)\vartheta_\pm^I \\ &\quad + \frac{35\pi}{k}i\left(\Lambda_{\pm I}^{(9/2)} + 4\Lambda_{\pm I}^{(7/2)'} - \frac{k}{60\pi}\psi_I^{\pm''}\right)(\vartheta_\pm^I)' - \frac{7}{4}i\left(\psi_I^{\pm'} - \frac{40\pi}{k}\Lambda_{\pm I}^{(7/2)}\right)(\vartheta_\pm^I)'' \\ &\quad - \frac{35}{12}i\psi_I^\pm(\vartheta_\pm^I)''' - \frac{1}{6}\left[\left(\mathcal{U}^\pm - \frac{1}{2}\tilde{\mathcal{L}}^{\pm''}\right)'' + \frac{144}{k}\left(\Lambda_\pm^{(6)} - \frac{49}{216}\tilde{\Lambda}_\pm^{(4)''}\right)\right]'\chi_\pm \\ &\quad - \frac{5}{6}\left[\left(\mathcal{U}^\pm - \frac{2}{3}\tilde{\mathcal{L}}^{\pm''}\right)'' + \frac{288}{5k}\Lambda_\pm^{(6)}\right]\chi_\pm' + \frac{14}{9}\left[\tilde{\mathcal{L}}^{\pm''} - \frac{27}{28}\mathcal{U}^\pm - \frac{21\pi}{k}\tilde{\Lambda}_\pm^{(4)}\right]'\chi_\pm'' \\ &\quad + \frac{7}{3}\left[\tilde{\mathcal{L}}^{\pm''} - \frac{3}{7}\mathcal{U}^\pm - \frac{28\pi}{3k}\tilde{\Lambda}_\pm^{(4)}\right]\chi_\pm''' + \frac{35}{18}\tilde{\mathcal{L}}^{\pm'}\chi_\pm^{(4)} + \frac{7}{9}\tilde{\mathcal{L}}^\pm\chi_\pm^{(5)} - \frac{k}{2432\pi}\chi_\pm^{(7)},\end{aligned}$$

where

$$\begin{aligned}\Lambda_{\pm IJ}^{(2)} &= \mathcal{J}_{IK}^\pm\mathcal{J}_{J}^{\pm K}, \\ \Lambda_{\pm IJ}^{(3)} &= \tilde{\mathcal{L}}^\pm\mathcal{J}_{IJ}^\pm - \Lambda_{\pm IJ}^{(2)'} - \mathcal{J}_{IK}^\pm(\mathcal{J}_J^{\pm K})' - \frac{10\pi}{k}\Lambda_{\pm I}^{(2)K}\mathcal{J}_{KJ}^\pm, \\ \Lambda_{\pm IJ}^{(4)} &= \delta_{IJ}(\tilde{\mathcal{L}}^\pm)^2 - \frac{50}{9}\left[\mathcal{J}_{IJ}^\pm\tilde{\mathcal{L}}^\pm - \frac{3}{2}\Lambda_{\pm IJ}^{(2)'} + \mathcal{J}_{IK}^\pm\mathcal{J}_J^{\pm K}\right]' - \frac{25}{9}\mathcal{J}_{IK}^{\pm'}\mathcal{J}_J^{\pm K'} \\ &\quad - \frac{500\pi}{9k}\left[\tilde{\mathcal{L}}^\pm\Lambda_{\pm IJ}^{(2)} - \frac{1}{2}\mathcal{J}^{\pm MN}(\mathcal{J}_{IM}^\pm\mathcal{J}_{NJ}^\pm)' - \mathcal{J}_{IM}^\pm(\mathcal{J}^{\pm MN}\mathcal{J}_{NJ}^\pm)'\right] \\ &\quad + \frac{2500\pi}{9k^2}\Lambda_{\pm I}^{(2)K}\Lambda_{\pm KJ}^{(2)}, \\ \tilde{\Lambda}_\pm^{(4)} &= (\tilde{\mathcal{L}}^\pm)^2,\end{aligned}$$

$$\begin{aligned}
\Lambda_{\pm}^{(6)} &= -\frac{7}{18}\mathcal{U}^{\pm}\tilde{\mathcal{L}}^{\pm} + \frac{295}{432}\left(\tilde{\mathcal{L}}^{\pm}\right)^2 + \frac{22}{27}\tilde{\mathcal{L}}^{\pm\prime\prime}\tilde{\mathcal{L}}^{\pm} + \frac{25}{12}i\psi_K^{\pm}\psi^{\pm K\prime} \\
&\quad - \frac{8\pi}{3k}\left[\left(\tilde{\mathcal{L}}^{\pm}\right)^3 - \frac{125}{16}i\mathcal{J}^{\pm KL}\psi_K^{\pm}\psi_L^{\pm}\right] . \\
\Lambda_{\pm I}^{(7/2)} &= \psi_K^{\pm}\mathcal{J}_I^{\pm K} , \\
\Lambda_{\pm I}^{(9/2)} &= \tilde{\mathcal{L}}^{\pm}\psi_I^{\pm} + \frac{5}{3}\psi_K^{\pm}\mathcal{J}_I^{\pm K\prime} - \frac{10}{3}\Lambda_{\pm I}^{(7/2)\prime} - \frac{50\pi}{3k}\psi_K^{\pm}\Lambda_{\pm I}^{(2)K} , \\
\Lambda_{\pm I}^{(11/2)} &= \frac{27}{23}\left(\tilde{\mathcal{L}}^{\pm\prime}\psi_I^{\pm} - \frac{260}{27}\mathcal{J}_I^{\pm K\prime}\psi_K^{\pm\prime} - \frac{370}{81}\mathcal{J}_I^{\pm K\prime}\psi_K^{\pm\prime\prime} - \frac{260}{81}\mathcal{J}_I^{\pm K\prime\prime}\psi_K^{\pm}\right) \\
&\quad + \frac{820\pi}{23k}\left(\tilde{\mathcal{L}}^{\pm}\Lambda_{\pm I}^{(7/2)} - \frac{25}{41}\Lambda_{\pm I}^{(2)K}\psi_K^{\pm\prime} + \frac{130}{123}\Lambda_{\pm K}^{(7/2)}\mathcal{J}_I^{\pm K\prime} + \frac{55}{123}\mathcal{J}_I^{\pm K}\Lambda_{\pm K}^{(7/2)\prime}\right) \\
&\quad + \frac{5000\pi^2}{23k^2}\Lambda_{\pm K}^{(7/2)}\Lambda_{\pm I}^{(2)K} .
\end{aligned}$$

Here the prime denotes derivative with respect to  $\varphi$ , and  $\chi_{\pm}^{(n)}$  denotes the  $n$ -th derivative of  $\chi_{\pm}$ .

## Appendix B. Poisson brackets of the canonical generators

The Poisson brackets of the asymptotic symmetry generators are given by

$$\begin{aligned}
[\mathcal{L}(\varphi), \mathcal{L}(\phi)]_{PB} &= -2\delta'(\varphi - \phi)\mathcal{L}(\varphi) - \delta(\varphi - \phi)\mathcal{L}'(\varphi) + \frac{k}{4\pi}\delta'''(\varphi - \phi) , \\
[\mathcal{L}(\varphi), \mathcal{U}(\phi)]_{PB} &= -4\delta'(\varphi - \phi)\mathcal{U}(\varphi) - 3\delta(\varphi - \phi)\mathcal{U}'(\varphi) , \\
[\mathcal{L}(\varphi), \mathcal{J}^{IJ}(\phi)]_{PB} &= -\mathcal{J}^{IJ}(\varphi)\delta'(\varphi - \phi) , \\
[\mathcal{L}(\varphi), \psi^{[I}(\phi)]_{PB} &= -\frac{5}{2}\delta'(\varphi - \phi)\psi^{[I}(\varphi) - \frac{3}{2}\delta(\varphi - \phi)\psi^{[I\prime}(\varphi) , \\
[\mathcal{U}(\varphi), \mathcal{J}^{IJ}(\phi)]_{PB} &= 0 , \\
[\mathcal{J}^{IJ}(\varphi), \mathcal{J}_{KL}(\phi)]_{PB} &= -4\delta_{[K}^{[I}\mathcal{J}_{L]}^{J]}(\varphi)\delta(\varphi - \phi) - \frac{k}{5\pi}\delta_{[K}^{[I}\delta_{L]}^{J]}\delta'(\varphi - \phi) , \quad (\text{B.1}) \\
[\mathcal{J}^{IJ}(\varphi), \psi^K(\phi)]_{PB} &= 2\delta^{K[I}\psi^{J]}(\varphi)\delta(\varphi - \phi) , \\
[\mathcal{U}(\varphi), \psi^I(\phi)]_{PB} &= \frac{1}{12}\left\{\left[\psi^{I\prime\prime\prime}(\varphi) - \frac{92\pi}{k}\left(\Lambda^{(9/2)I}(\varphi) + \frac{210}{23}\Lambda^{(7/2)I\prime}(\varphi)\right)\right]' \right. \\
&\quad \left. - \frac{92\pi}{k}\Lambda^{(11/2)I}(\varphi)\right\}\delta(\varphi - \phi) + \frac{7}{12}\left(\psi^{I\prime\prime\prime}(\varphi) - \frac{60\pi}{k}\Lambda^{(9/2)I}(\varphi) \right. \\
&\quad \left. - \frac{240\pi}{k}\Lambda^{(7/2)I\prime}(\varphi)\right)\delta'(\varphi - \phi) + \frac{35}{12}\psi^I(\varphi)\delta'''(\varphi - \phi) \\
&\quad + \frac{7}{4}\left(\psi^{I\prime} - \frac{40\pi}{k}\Lambda^{(7/2)I}(\varphi)\right)\delta''(\varphi - \phi) ,
\end{aligned}$$



$$\begin{aligned}
[\mathcal{U}(\varphi), \mathcal{U}(\phi)]_{PB} &= \frac{5}{6} \left[ \left( \mathcal{U}(\varphi) - \frac{2}{3} \tilde{\mathcal{L}}''(\varphi) \right)'' + \frac{288\pi}{5k} \Lambda^{(6)}(\varphi) \right] \delta'(\varphi - \phi) \\
&\quad + \frac{1}{6} \left[ \left( \mathcal{U}(\varphi) - \frac{1}{2} \tilde{\mathcal{L}}''(\varphi) - \frac{98\pi}{3k} \tilde{\Lambda}^{(4)}(\varphi) \right)'' + \frac{144\pi}{k} \Lambda^{(6)}(\varphi) \right]' \delta(\varphi - \phi) \\
&\quad + \frac{3}{2} \left( \mathcal{U}(\varphi) - \frac{28}{27} \tilde{\mathcal{L}}''(\varphi) + \frac{196\pi}{9k} \tilde{\Lambda}^{(4)}(\varphi) \right)' \delta''(\varphi - \phi) \\
&\quad + \left( \mathcal{U}(\varphi) - \frac{7}{3} \tilde{\mathcal{L}}''(\varphi) + \frac{196\pi}{9k} \tilde{\Lambda}^{(4)}(\varphi) \right) \delta'''(\varphi - \phi) \\
&\quad - \frac{35}{18} \tilde{\mathcal{L}}'(\varphi) \delta^{(4)}(\varphi - \phi) - \frac{7}{9} \tilde{\mathcal{L}}(\varphi) \delta^{(5)}(\varphi - \phi) + \frac{k}{144\pi} \delta^{(7)}(\varphi - \phi) , \\
i[\psi^I(\varphi), \psi^J(\phi)]_{PB} &= \delta(\phi - \varphi) \left[ \delta^{IJ} \mathcal{U}(\varphi) - \frac{1}{2} \delta^{IJ} \tilde{\mathcal{L}}''(\varphi) + \frac{5}{6} \mathcal{J}^{IJ''}(\varphi) + \frac{3\pi}{k} \Lambda^{(4)(IJ)}(\varphi) \right. \\
&\quad + \frac{25\pi}{k} \Lambda^{(2)JI'}(\varphi) + \frac{50\pi}{3k} \Lambda^{(3)JI'}(\varphi) \left. \right] + \frac{10}{3} \mathcal{J}^{IJ}(\varphi) \delta'''(\phi - \varphi) \\
&\quad + \delta'(\phi - \varphi) \left[ \frac{10}{3} \mathcal{J}^{IJ''}(\varphi) - \frac{5}{3} \delta^{IJ} \tilde{\mathcal{L}}'(\varphi) - \frac{100\pi}{3k} \Lambda^{(3)[IJ]}(\varphi) \right. \\
&\quad + \frac{50\pi}{k} \Lambda^{(2)JI'}(\varphi) \left. \right] + \frac{k}{12\pi} \delta^{IJ} \delta''''(\phi - \varphi) \\
&\quad + \delta''(\phi - \varphi) \left[ 5 \mathcal{J}^{IJ'}(\varphi) - \frac{5}{3} \delta^{IJ} \tilde{\mathcal{L}}(\varphi) + \frac{50\pi}{k} \Lambda^{(2)IJ}(\varphi) \right] , \tag{B.2}
\end{aligned}$$

so that once expanded in Fourier modes, the algebra corresponds to the one in eqs. (10)

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